GOAL

Use properties of midsegments and write coordinate proofs.

Vocabulary

A **midsegment of a triangle** is a segment that connects the midpoints of two sides of the triangle.

A **coordinate proof** involves placing geometric figures in a coordinate plane. When you use variables to represent the coordinates of a figure in a coordinate proof, the results are true for all figures of that type.

Theorem 5.1 Midsegment Theorem: The segment connecting the midpoints of two sides of a triangle is parallel to the third side and is half as long as that side.

EXAMPLE 1

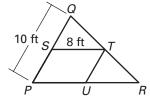
Use the Midsegment Theorem to find lengths

In the diagram, \overline{ST} and \overline{TU} are midsegments of $\triangle PQR$. Find PR and TU.

Solution

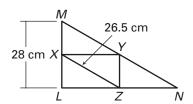
$$PR = 2 \cdot ST = 2(8 \text{ ft}) = 16 \text{ ft}$$

$$TU = \frac{1}{2} \cdot QP = \frac{1}{2}(10 \text{ ft}) = 5 \text{ ft}$$



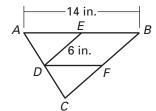
Exercises for Example 1

1. In the diagram, \overline{XZ} and \overline{ZY} are midsegments of $\triangle LMN$. Find MN and ZY.



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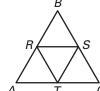
2. In the diagram, \overline{ED} and \overline{DF} are midsegments of $\triangle ABC$. Find DF and BC.



EXAMPLE 2

Use the Midsegment Theorem

In the diagram at the right, $\overline{SB} \cong \overline{SC}$, $\overline{RS} \parallel \overline{AC}$, and $RS = \frac{1}{2}AC$. Show that R is the midpoint of \overline{BA} .



Solution

Because $\overline{SB} \cong \overline{SC}$, S is the midpoint of \overline{BC} . Because $\overline{RS} \parallel \overline{AC}$ and $RS = \frac{1}{2}AC$, \overline{RS} is a midsegment of $\triangle ABC$ by definition. By the Midsegment Theorem, R is the midpoint of \overline{BA} .

EXAMPLE 3

Place a figure in a coordinate plane

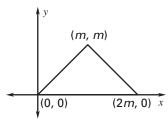
Place each figure in a coordinate plane in a way that is convenient for finding side lengths. Assign coordinates to each vertex.

- **a.** An isosceles triangle
- **b.** A square

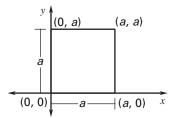
Solution

It is easy to find lengths of horizontal and vertical segments and distances from (0, 0), so place one vertex at the origin and one or more sides on an axis.

a. Let 2*m* represent the length of the base of the isosceles triangle. The coordinates of the vertex opposite the base is (*m*, *m*), which makes each of the legs congruent.



b. Let *a* represent the side length of the square.



Exercises for Examples 2 and 3

- **3.** In Example 2, if *T* is the midpoint of \overline{AC} , what do you know about \overline{ST} ?
- **4.** A rectangle has vertices (0, 0), (j, 0), and (j, k). Find the fourth vertex.