GOAL Use medians and altitudes of triangles.

Vocabulary

A **median of a triangle** is a segment from a vertex to the midpoint of the opposite side.

The point of concurrency of the three medians of a triangle is called the **centroid**, and is always inside the triangle.

An **altitude of a triangle** is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.

The point at which the lines containing the three altitudes of a triangle intersect is called the **orthocenter** of the triangle.

Theorem 5.8 Concurrency of Medians of a Triangle: The medians of a triangle intersect at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

Theorem 5.9 Concurrency of Altitudes of a Triangle: The lines containing the altitudes of a triangle are concurrent.

EXAMPLE 1 Use the centroid of a triangle

In \triangle ABC, D is the centroid and BD = 12. Find DG and BG.

Solution

$$BD = \frac{2}{3}BG$$
 Concurrency of Medians of a Triangle Theorem

$$12 = \frac{2}{3}BG$$
 Substitute 12 for BD .

$$18 = BG$$
 Multiply each side by the reciprocal, $\frac{3}{2}$.

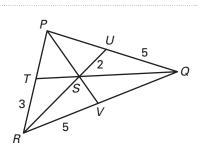
Then
$$DG = BG - BD = 18 - 12 = 6$$
.

So,
$$DG = 6$$
 and $BG = 18$.

Exercises for Example 1

In $\triangle PQR$, S is the centroid, $\overline{PQ} \cong \overline{PQ}$, UQ = 5, TR = 3, and SU = 2.

- **1.** Find *RU* and *RS*.
- **2.** Find the perimeter of $\triangle PQR$.



LESSON 5.4 **Study Guide** continued For use with pages 318–327

EXAMPLE 2 Find the centroid of a triangle

The vertices of \triangle *ABC* are A(0, 0), B(4, 10), and C(8, 2). Find the coordinates of the centroid P of \triangle *ABC*.

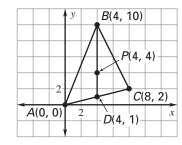
Solution

Sketch $\triangle ABC$. Then use the Midpoint Formula to find the midpoint D of \overline{AC} and sketch median \overline{BD} .

$$D\left(\frac{0+8}{2}, \frac{0+2}{2}\right) = D(4, 1)$$

The centroid is two thirds of the distance from each vertex to the midpoint of the opposite side.

The distance from vertex B(4, 10) to D(4, 1) is 10 - 1 = 9 units. So, the centroid is $\frac{2}{3}(9) = 6$ units down from B on \overline{BD} .

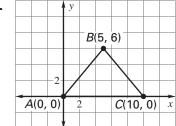


The coordinates of the centroid P are (4, 10 - 6) or (4, 4).

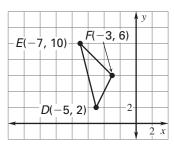
Exercises for Example 2

Find the coordinates of the centroid of the triangle with the given vertices.

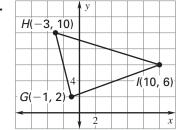
3.



4.



5.



6.

