Study Guide
For use with pages 408–415

GOAL **Perform dilations.**

Vocabulary

A **dilation** is a transformation that stretches or shrinks a figure to create a similar figure.

In a dilation, a figure is enlarged or reduced with respect to a fixed point called the **center of dilation.**

The **scale factor of a dilation** is the ratio of a side length of the image to the corresponding side length of the original figure.

Coordinate Notation for a Dilation:
You can describe a dilation with respect to the origin with the notation $(x, y) \rightarrow (kx, ky)$, where k is the scale factor.

If $0 < k < 1$, the dilation is a **reduction.** If $k > 1$, the dilation is an **enlargement.**

EXAMPLE 1 ## Draw a dilation

Draw a dilation of quadrilateral *ABCD* with vertices *A*(0, 3), *B*(2, 3), *C*(3, 1), and *D*(2, 0). Use a scale factor of 3.

Solution

First draw *ABCD*. Find the dilation of each vertex by multiplying its coordinates by 3. Then draw the dilation.
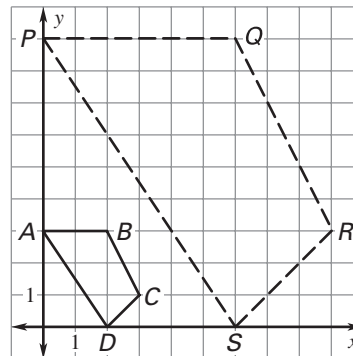
$$(x, y) \rightarrow (3x, 3y)$$

$$A(0, 3) \rightarrow P(0, 9)$$

$$B(2, 3) \rightarrow Q(6, 9)$$

$$C(3, 1) \rightarrow R(9, 3)$$

$$D(2, 0) \rightarrow S(6, 0)$$

Exercises for Example 1

Find the coordinates of *L*, *M*, and *N* so that △*LMN* is a dilation of △*PQR* with a scale factor of *k*. Sketch △*PQR* and △*LMN*.

1. $P(-2, 3), Q(2, 1), R(-2, -2); k = 2$

2. $P(5, 0), Q(5, 5), R(10, 10); k = \frac{1}{5}$

3. $P(0, 4), Q(2, 1), R(-4, 2); k = \frac{3}{2}$

4. $P(-3, 2), Q(-1, 3), R(1, 2); k = 3$

EXAMPLE 2   Find a scale factor

Photograpy A digital photograph has the height shown in the diagram. You want to reduce the size of the photograph to the height shown. What is the scale factor of the reduction?

8.5 in.

3 in.

Original   Reduction

Solution

The scale factor is the ratio of a side length of the reduced photo to a side length of the original photo, or $\dfrac{3 \text{ in.}}{8.5 \text{ in.}}$. In simplest form, the scale factor is $\dfrac{6}{17}$.
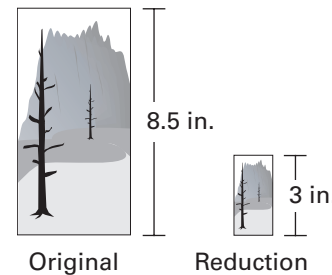
EXAMPLE 3   Find a point on a dilation

You want to create a quadrilateral *ABCD* that is similar to quadrilateral *FGHJ*. What are the coordinates of *D*?

Solution

Determine if *ABCD* is a dilation of *FGHJ* by checking whether the same scale factor can be used to obtain *A*, *B*, and *C* from *F*, *G*, and *H*.

$$(x, y) \rightarrow (kx, ky)$$

$$F(-1, 0) \rightarrow A(-4, 0) \qquad k = 4$$

$$G(1, -1) \rightarrow B(4, -4) \qquad k = 4$$

$$H(0, -3) \rightarrow C(0, -12) \qquad k = 4$$

Because *k* is the same in each case, the image is a dilation with a scale factor of 4. So, you can use the scale factor to find the image *D* of point *J*.

$$J(-2, -4) \rightarrow D(4 \cdot -2, 4 \cdot -4) = D(-8, -16)$$

Exercises for Examples 2 and 3

5. A digital photograph is 7.5 centimeters wide. You want to reduce its size proportionally so the new width is 2.5 centimeters. What is the scale factor of the reduction?

You want to create a pentagon *ABCDE* that is similar to pentagon *FGHJK* in the diagram at the right.

6. What is the scale factor?

7. What are the coordinates of *D* and *E*?

LESSON 6.7