Name ______________________ Date ____________

LESSON 9.2

Study Guide

For use with pages 580–587

GOAL **Perform translations using matrix operations.**

Vocabulary

A **matrix** is a rectangular arrangement of numbers in rows and columns.

An **element** is a number in a matrix.

The **dimensions of a matrix** are the numbers of rows and columns, written as rows × columns.

EXAMPLE 1 Add and subtract matrices

1. $\begin{bmatrix} 4 & -4 \\ 5 & -7 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 2 & -5 \end{bmatrix} = \begin{bmatrix} 4+0 & -4+1 \\ 5+2 & -7+(-5) \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 7 & -12 \end{bmatrix}$

2. $\begin{bmatrix} 7 & 9 & 6 \\ 5 & 10 & 0 \end{bmatrix} - \begin{bmatrix} 3 & -5 & 2 \\ 5 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 7-3 & 9-(-5) & 6-2 \\ 5-5 & 10-(-2) & 0-5 \end{bmatrix}$

$= \begin{bmatrix} 4 & 14 & 4 \\ 0 & 12 & -5 \end{bmatrix}$

Exercises for Example 1

Add or subtract.

1. $\begin{bmatrix} -5 & 9 \end{bmatrix} + \begin{bmatrix} 4 & -7 \end{bmatrix}$

2. $\begin{bmatrix} 3 & -6 \\ 5 & -8 \end{bmatrix} - \begin{bmatrix} 4 & 5 \\ 9 & 6 \end{bmatrix}$

EXAMPLE 2 Represent a translation using matrices

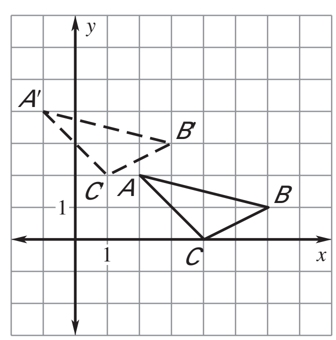
The matrix $\begin{bmatrix} 2 & 6 & 4 \\ 2 & 1 & 0 \end{bmatrix}$ represents $\triangle ABC$. Find the image matrix that represents the translation of $\triangle ABC$ 3 units left and 2 units up. Then graph $\triangle ABC$ and its image.

Solution

The translation matrix is $\begin{bmatrix} -3 & -3 & -3 \\ 2 & 2 & 2 \end{bmatrix}$.

Add this to the polygon matrix for the preimage to find the image matrix.

$$\underset{\text{Translation Matrix}}{\begin{bmatrix} -3 & -3 & -3 \\ 2 & 2 & 2 \end{bmatrix}} + \underset{\text{Polygon Matrix}}{\overset{\begin{matrix} A & B & C \end{matrix}}{\begin{bmatrix} 2 & 6 & 4 \\ 2 & 1 & 0 \end{bmatrix}}} = \underset{\text{Image Matrix}}{\overset{\begin{matrix} A' & B' & C' \end{matrix}}{\begin{bmatrix} -1 & 3 & 1 \\ 4 & 3 & 2 \end{bmatrix}}}$$



LESSON 9.2

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EXAMPLE 3 Multiply matrices

Multiply $\begin{bmatrix} 2 & 3 \\ 6 & 0 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ -2 & 7 \end{bmatrix}$.

Solution

The matrices are both 2×2, so their product is defined. Use the following steps to find the elements of the product matrix.

STEP 1 **Multiply** the numbers in the 1st row of the 1st matrix by the numbers in the 1st column of the 2nd matrix. Put the result in the 1st row, 1st column of the product matrix.

$$\begin{bmatrix} 2 & 3 \\ 6 & 0 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} 2(4) + 3(-2) & ? \\ ? & ? \end{bmatrix}$$

STEP 2 **Multiply** the numbers in the 1st row of the 1st matrix by the numbers in the 2nd column of the 2nd matrix. Put the result in the 1st row, 2nd column of the product matrix.

$$\begin{bmatrix} 2 & 3 \\ 6 & 0 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} 2(4) + 3(-2) & 2(-5) + 3(7) \\ ? & ? \end{bmatrix}$$

STEP 3 **Multiply** the numbers in the 2nd row of the 1st matrix by the numbers in the 1st column of the 2nd matrix. Put the result in the 2nd row, 1st column of the product matrix.

$$\begin{bmatrix} 2 & 3 \\ 6 & 0 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} 2(4) + 3(-2) & 2(-5) + 3(7) \\ 6(4) + 0(-2) & ? \end{bmatrix}$$

STEP 4 **Multiply** the numbers in the 2nd row of the 1st matrix by the numbers in the 2nd column of the 2nd matrix. Put the result in the 2nd row, 2nd column of the product matrix.

$$\begin{bmatrix} 2 & 3 \\ 6 & 0 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} 2(4) + 3(-2) & 2(-5) + 3(7) \\ 6(4) + 0(-2) & 6(-5) + 0(7) \end{bmatrix}$$

STEP 5 **Simplify** the product matrix.

$$\begin{bmatrix} 2(4) + 3(-2) & 2(-5) + 3(7) \\ 6(4) + 0(-2) & 6(-5) + 0(7) \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 24 & -30 \end{bmatrix}$$

Exercises for Examples 2 and 3

3. The matrix $\begin{bmatrix} 0 & 1 & 5 & 6 \\ 1 & -2 & 0 & 2 \end{bmatrix}$ represents quadrilateral *JKLM*. Write the translation matrix that represents the translation of *JKLM* 3 units right and 4 units down. Then graph quadrilateral *JKLM* and its image.

Multiply.

4. $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 9 \\ -3 & 5 \end{bmatrix}$

5. $\begin{bmatrix} 6 & 2 \end{bmatrix}\begin{bmatrix} -8 \\ -7 \end{bmatrix}$

6. $\begin{bmatrix} -6 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 4 & 1 \end{bmatrix}$