GOAL Rotate figures about a point.

#### Vocabulary

A **rotation** is a transformation in which a figure is turned about a fixed point.

The **center of rotation** is the fixed point in which a figure is turned about.

The **angle of rotation** is formed from rays drawn from the center of rotation to a point and its image.

**Theorem 9.3 Rotation Theorem:** A rotation is an isometry.

#### Rotate a figure using the coordinate rules **EXAMPLE 1**

Graph quadrilateral RSTU with vertices R(1, -1), S(3, -1), T(3, -5), and U(0, -4). Then rotate the quadrilateral 270° about the origin.

**Solution** 

$$(a, b) \rightarrow (b, -a)$$

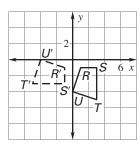
$$R(1,-1) \to R'(-1,-1)$$

$$S(3,-1) \rightarrow S'(-1,-3)$$

$$T(3, -5) \rightarrow T'(-5, -3)$$
  
 $U(0, -4) \rightarrow U'(-4, 0)$ 

Graph the image 
$$R'S'T'U'$$
.

Graph the image R'S'T'U'.



#### **Exercise for Example 1**

**1.** Graph  $\triangle JKL$  with vertices J(4, 1), K(5, 4), and L(7, 0). Rotate the triangle  $90^{\circ}$ about the origin.

LESSON 9.4

# **Study Guide** continued For use with pages 598–605

## **EXAMPLE 2** Use matrices to rotate a figure

Trapezoid *EFGH* has vertices E(-4, 1), F(-4, 3), G(0, 3), and H(1, 1). Find the image matrix for a 180° rotation of *EFGH* about the origin. Graph *EFGH* and its image.

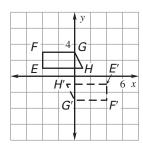
**Solution** 

STEP 1 Write the polygon matrix: 
$$\begin{bmatrix} E & F & G & H \\ -4 & -4 & 0 & 1 \\ 1 & 3 & 3 & 1 \end{bmatrix}$$

**STEP 2** Multiply by the matrix for a 180° rotation.

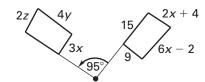
$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} E & F & G & H \\ -4 & -4 & 0 & 1 \\ 1 & 3 & 3 & 1 \end{bmatrix} = \begin{bmatrix} E' & F' & G' & H' \\ 4 & 4 & 0 & -1 \\ -1 & -3 & -3 & -1 \end{bmatrix}$$

**STEP 3 Graph** the preimage EFGH and the image E'F'G'H'.



### **EXAMPLE3** Use Theorem 9.3

Find the value of *y* in the rotation of the quadrilateral.



#### **Solution**

By Theorem 9.3, the rotation is an isometry, so corresponding side lengths are equal. Then 3x = 9, so x = 3. Now set up an equation to solve for y.

$$4v = 6x - 2$$

Corresponding lengths in an isometry are equal.

$$4y = 6(3) - 2$$

Substitute 3 for *x*.

$$v = 4$$

Solve for *y*.

### **Exercises for Examples 2 and 3**

Use the trapezoid *EFGH* in Example 2. Find the image matrix after the rotation about the origin. Graph the image.

**5.** In Example 3, find the value of *z* in the rotation of the quadrilateral.