Name ______________________________ Date ____________________

LESSON 9.5

Study Guide

For use with pages 607–615

GOAL **Perform combinations of two or more transformations.**

Vocabulary

A **glide reflection** is a transformation in which every point P is mapped to a point P'' by the following steps. (1) A translation maps P to P'. (2) A reflection in a line k parallel to the direction of the translation maps P' to P''.

A **composition of transformations** is the result of two or more transformations that are combined to form a single transformation.

Theorem 9.4 Composition Theorem: The composition of two (or more) isometries is an isometry.

Theorem 9.5 Reflections in Parallel Lines Theorem: If lines k and m are parallel, then a reflection in line k followed by a reflection in line m is the same as a translation. If P'' is the image of P, then:

1. $\overline{PP''}$ is perpendicular to k and m, and
2. $PP'' = 2d$, where d is the distance between k and m.

Theorem 9.6 Reflections in Intersecting Lines Theorem: If lines k and m intersect at point P, then a reflection in k followed by a reflection in m is the same as a rotation about point P. The angle of rotation is $2x°$, where $x°$ is the measure of the acute or right angle formed by k and m.

EXAMPLE 1

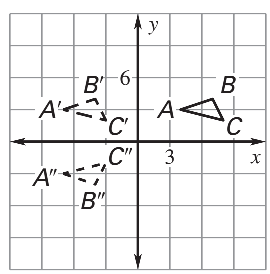
Find the image of a glide reflection

The vertices of $\triangle ABC$ are $A(4, 3)$, $B(7, 4)$, and $C(8, 2)$. Find the image of $\triangle ABC$ after the glide reflection.

Translation: $(x, y) \rightarrow (x - 11, y)$

Reflection: in the x-axis

Solution



Begin by graphing $\triangle ABC$. Then, graph $\triangle A'B'C'$ after a translation 11 units left. Finally, graph $\triangle A''B''C''$ after a reflection in the x-axis.

Exercises for Example 1

1. Suppose $\triangle ABC$ in Example 1 is translated 5 units down, then reflected in the y-axis. What are the coordinates of the vertices of the image?

2. In Example 1, describe a glide reflection from $\triangle A''B''C''$ to $\triangle ABC$.

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EXAMPLE 2 Find the image of a composition

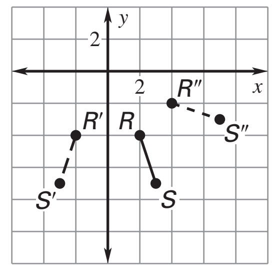
The endpoints of $\overline{RS}$ are $R(2, -4)$ and $S(3, -7)$. Graph the image of $\overline{RS}$ after the composition.

Reflection: in the y-axis

Rotation: 90° about the origin

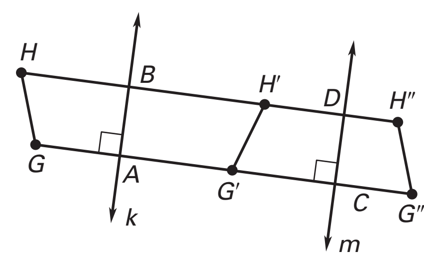
Solution

Graph $\overline{RS}$. Reflect $\overline{RS}$ in the y-axis. $\overline{R'S'}$ has endpoints $R'(-2, -4)$ and $S'(-3, -7)$. Rotate $\overline{R'S'}$ 90° about the origin. $\overline{R''S''}$ has endpoints $R''(4, -2)$ and $S''(7, -3)$.



EXAMPLE 3 Use Theorem 9.5

In the diagram, a reflection in line k maps $\overline{GH}$ to $\overline{G'H'}$. A reflection in line m maps $\overline{G'H'}$ to $\overline{G''H''}$. Also, $HB = 10$ and $DH'' = 5$.



a. Name any segments congruent to each segment: $\overline{HG}$, $\overline{HB}$, $\overline{GA}$.

b. Does $AC = BD$? *Explain.*

c. What is the length of $\overline{GG''}$?

Solution

a. $\overline{HG} \cong \overline{H'G'}$, $\overline{HG} \cong \overline{H''G''}$, $\overline{HB} \cong \overline{H'B}$, $\overline{GA} \cong \overline{G'A}$

b. Yes, $AC = BD$ because $\overline{GG''}$ and $\overline{HH''}$ are perpendicular to both k and m, so $\overline{BD}$ and $\overline{AC}$ are opposite sides of a rectangle.

c. By the properties of reflections, $H'B = 10$ and $H'D = 5$. Theorem 9.5 implies that $GG'' = HH'' = 2 \cdot BD$, so the length of $\overline{GG''}$ is $2(10 + 5)$, or 30 units.

Exercises for Examples 2 and 3

3. Graph $\overline{RS}$ from Example 2. Do the rotation first, followed by the reflection. Does the order of the transformations matter? *Explain.*

4. In Example 3 part (c), *explain* how you know that $GG'' = HH''$.

5. In Example 3, $\overline{HG}$ is reflected in line k, then in line m. *Describe* a single transformation that maps HG to $\overline{H''G''}$.

6. In Example 3, the distance between line k and m is 15 units. What is the distance between H and H''? If you draw $\overline{HH'}$, what is the relationship with line k?